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Ginette Saracco, Philippe Tchamitchian. Retrievial of a Time-Dependent Source in an Acoustic Propagation Problem. Inverse Problems & Theoretical Image, SPRINGER-VERLAG, 1990, pp.207-211. hal-00157751

HAL Id: hal-00157751
https://hal.science/hal-00157751

Submitted on 27 Jun 2007

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Inverse Methods in Action, PC Sabatier Edt
Inverse Problems, Nov. 27^{th}-Dec. 1^{st}, 1989
SPRINGER-VERLAG, p.207-211, 1990

Retrieval of a Time-Dependent Source in an Acoustic Propagation Problem

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Abstract : Consider two media separated by a plane interface, a time-dependent source F(t) at a point S in the first medium (of the lower celerity), receivers in the second medium, located at a large radial distance from the source. Thanks to a diffusion-like behaviour of the transmitted wave, we are able to retrieve the term F(t), under high frequencies hypothesis in an elementary way.

1. About homogeneity and cancellation

We begin with the Poisson semi-group of operators

$$P_y = e^{-y\Lambda},$$

where $y > 0$ and $\Lambda = \sqrt{-\Delta}$, acting on $L^2(\mathbb{R}^n)$. Then, we define

$$Q_y = -y \frac{\partial}{\partial y} P_y .$$

Because of the fact that P_y is an approximation of the identity, which means that

$$I = \lim_{y \to 0} P_y \quad \text{and} \quad 0 = \lim_{y \to +\infty} P_y$$

in the strong topology of $L^2(\mathbb{R}^n)$, we deduce the following decomposition of the identity :

$$I = \int_0^{+\infty} Q_y \frac{dy}{y} . \tag{1}$$

This formula is known as the Littlewood-Paley decomposition, and has extensively been studied in harmonic analysis since the 30's. See for example the classical treatise by Zygmund [4]. For the amateurs, we mention that is in fact the ancestor of the so-called wavelet transform. But we do not want to go further into this matter.

What we would like to point out here is that analogous formulas to (1) can be obtained, and may be useful, in some physical situations which a priori have nothing to do with diffusion.

Before stating what we have in mind, let us have a closer look at (1), on the Fourier side.

(1) may be interpreted as a way of analysing signals with a continuous chain of filters associated to the operators Q_y. More precisely, if we denote these filters by $Q_y(\omega)$, then :

$$Q_y(\omega) = y\,|\,\omega\,|\,e^{-y|\omega|}\,.$$

Now, it becomes apparent why (1) is true. It is because of the following two basic properties of $Q_y(\omega)$:

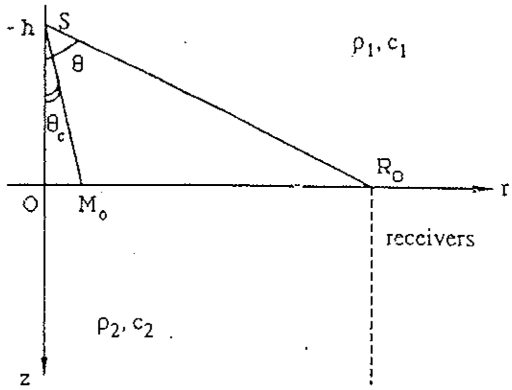
a. an homogeneity property : $Q_y(\omega)$ depends effectively on the product $y\,|\omega|$. It is centered around the sphere in $\mathbb{R}^n$ of radius $\frac{1}{y}$, with a bandwidth $\Delta Q_y = \frac{c_n}{y}$, where c_n is a constant (varying with your normalisation). Notice that $\frac{dy}{y}$ is the dilation-invariant measure on $\mathbb{R}^+$.

b. a cancellation property : $Q_y(\omega)$ cancels at the origin. Such a property is necessary in order to insure the convergence at $y = 0$ of the integral in (1). A contrario, the same integral with P_y instead of Q_y would strongly diverge.

We now turn our attention to a seemingly very different problem : the propagation through a plane interface.

2. An asymptotic formula for retrieving a time-dependent source

The physical situation is the classical one : two infinite homogeneous and isotropic media are separated by a plane interface. We think to air and water for numerical and experimental purposes [3]. An acoustical point source is emitting in the first medium (air), at height h, and receivers are located in the second one (water), at a large radial distance r from the source, and on a vertical straight line starting from the interface at the point R_o :



$\frac{\rho_2}{\rho_1} = m \sim 800$ (densities)

$\frac{c_1}{c_2} = n \sim 0.23$ (celerities)

$\theta_c = \arcsin n \sim 13°$ (critical angle)

Fig. 1

Here, "large radial distance" means that $\theta = \mathrm{arctg}\,\frac{r}{h}$ is greater than the critical angle θ_c (that is to say, $\frac{r}{\sqrt{r^2+h^2}} > n$).

If F(t) is the acoustic potential of the source, and if $\psi_2\,(r,z,t)$ is the transmitted potential at a point (r,z), $z > 0$, and at time t, the associated pressure $P_2\,(r,z,t)$ is

$$P_2\,(r,z,t) = \rho_2 \frac{\partial}{\partial_y} \psi_2\,(r,z,t)\,.$$

We recall that (see Landau and Lifshitz [2]) :

$$\psi_2\,(r,z,t) = \int_R e^{-i\omega t}\,\hat{F}(\omega)\; i\int_0^{+\infty} e^{i(k_1 h + k_2 z)}\, J_o(kr)\, \frac{1}{m\,k_1+k_2}\, k\,dk\,d\omega\,,$$

where $k_j = \sqrt{\left(\frac{\omega}{c_j}\right)^2 - k^2}\,.$

We suppose that the source is of a high frequency nature : $\hat{F}(\omega) \neq 0$ only if $|\omega| \geq \omega_{min}$, and $\frac{r\,\omega_{min}}{c_1} >> 1$.

Then, we have the following asymptotic formula :

$$\int_0^{+\infty} P_2\,(r,z,t)\,dz \sim C_1(r,h)\;\; F(t-t_1) + C_2(r,h)\;\; F(t-t_2) + C_3(r,h)\;\; \mathcal{H}F(t-t_2) \qquad (2)$$

where

$\mathcal{H}$ is the Hilbert transform ,

$t_1 = \dfrac{h\,\sqrt{1-n^2}+nr}{c_1}$,

$t_2 = \dfrac{\sqrt{r^2+h^2}}{c_1}$,

$C_j\,(r,h)$ are constants depending only on the positions of the source and the receivers.

t_1 is the time for a way to travel over the path S M_o R_o with velocity c_1 on S M_o and c_2 on M_o R_o, and t_2 corresponds to the path S R_o with velocity c_1 (see fig. 1).

The presence of the Hilbert transform is due to the condition $\frac{r}{\sqrt{r^2+h^2}} > n$. It disappears in the opposite case, where a similar (and much less interesting, at least for an air-water interface) formula holds :

$$\int_0^{+\infty} P_2\,(r,z,t)\,dt \sim C(r,h)\,F(t - t_2). \qquad (2')$$

If we take into consideration the fact that $m = \frac{P_2}{P_1} \approx 800$, we have for the constants in (2) the following simplified expressions :

$$C_1\,(r,h) = \frac{\rho_1}{(1-n^2)^{1/4}\,[r(r\sqrt{1-n^2} - hn)]^{1/2}} + 0\left(\frac{1}{m}\right)\,, \qquad C_2\,(r,h) = 0\left(\frac{1}{m}\right)\,,$$

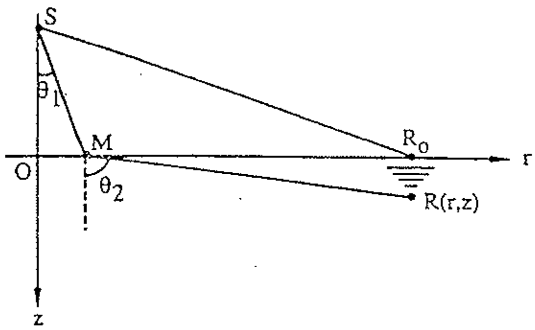
$$C_3\,(r,h) = \frac{\rho_1}{\left[\frac{r^2}{r^2+h^2} - n^2\right]^{1/2} r} + 0\left(\frac{1}{m}\right)\,.$$

We are not going to enter in the details of the proof of the formula (2), which is straightforward and typical of asymptotic arguments (one could ask whether such a kind of argument is necessary or not, the answer is yes, for the result is simply false for arbitrary sources).

We prefer to conclude with a comparison between formulas (1) and (2).

Let us remark however that, once (2) is valid and the position of the source is known, it is easy to retrieve the source term F(t). Maybe it is also possible to find the coordinates of the point source from (2), but this is still under investigations.

3. Link between the formulas (1) and (2)



$$\sin\ \theta_1 = n\ \sin\ \theta_2$$

Fig. 2

Looking at P_2 (r,z,t) as the result of a filtering process applied on F(t), we denote by $\Gamma(r,z,\omega)$ the associated impulse response.

$\Gamma(r,z,\omega)$ is nothing but the transmitted pressure at the point R(r,z) when the source is harmonic, with frequency ω. It is usual, following Brekhovskikh [1], to decompose it in two contributions, under asymptotic conditions :

$$\Gamma(r,z,\omega) = \Gamma_{geo}\,(r,z,\omega) + \Gamma_{lat}\,(r,z,\omega).$$

Γ_{geo} is given by the approximation of the geometrical acoustics (see fig. 2) :

$$\Gamma_{geo}\,(r,z,\omega) = \gamma_{geo}\,(r,z,h)\ i\,\omega\ e^{i\omega\left(\frac{SM}{c_1} + \frac{MR}{c_2}\right)}, \qquad (3)$$

while Γ_{lat} is specific of the case when $n = \frac{c_1}{c_2} < 1$:

$$\Gamma_{lat}\,(r,z,\omega) = \gamma_{lat}\,(r,h)\ e^{i\omega\frac{\sqrt{r^2+h^2}}{c_1}}\ i\omega\, e^{-\alpha|\omega|\frac{z}{c_2}}, \qquad (4)$$

$$\alpha = \sqrt{\frac{r^2}{r^2+h^2} - n^2}\,.$$

Looking at these expressions, we may understand why (2) is valid.

Consider at first Γ_{lat}, which is the main term, at least near the interface (see Saracco [3]). It is usually said that Γ_{lat} corresponds to a wave propagating in the air along the path S R_0 and then diffusing in the water (fig. 2). The term $e^{-\alpha|\omega|\frac{z}{c_1}}$ in (4) is typical of a diffusion process, while $e^{i\omega\frac{\sqrt{r^2+h^2}}{c_1}}$ is only a delay term. Γ_{lat} (r,z,ω) appears to be very much like $Q_y(\omega)$ in §1, with $\alpha\frac{z}{c_1}$ playing the role of y.

What about Γ_{geo} ? It is different in nature from Γ_{lat}, and associated to a wave propagating in air and in water along the path SMR (fig. 2). But it still has the properties of homogeneity and cancellation we described in § 1, at least again near the interface. In fact, if $\frac{z}{r}$ is small, we have

$$\Gamma_{geo}(r,z,\omega) \sim \gamma'_{geo}(r,h)\; e^{i\omega\frac{h\sqrt{1-n^2}+rn}{c_1}}\; i\omega z\, e^{i\frac{\omega z^2}{c_1\ell}}\,, \tag{5}$$

where $\ell = \frac{r}{n} - \frac{h}{\sqrt{1-n^2}}$. The cancellation property is already clear in (3), and the homogeneity is apparent in (5).

These remarks explain why the formulas (1) and (2) are analogous.

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